

LECTURE: 5-4 INDEFINITE INTEGRALS AND THE NET CHANGE THEOREM

Question: What does it mean to say $F(x)$ is an anti-derivative of $f(x)$? In other words, how would you check that $\int f(x)dx = F(x)$?

Example 1: Verify by differentiation that $\int x \cos x dx = x \sin x + \cos x + C$ is correct.

Indefinite Integrals

$$\int f(x)dx = F(x) \text{ means } \underline{\hspace{2cm}}$$

All the indefinite integrals you (should) already know:

- $\int x^n dx =$
- $\int \sin x dx =$
- $\int \cos x dx =$
- $\int \sec^2 x dx =$
- $\int \csc^2 x dx =$
- $\int \sec x \tan x =$
- $\int \csc x \cot x =$
- $\int \frac{1}{x} dx$
- $\int e^x dx$
- $\int a^x dx$
- $\int \frac{1}{\sqrt{1-x^2}} dx$
- $\int \frac{1}{1+x^2} dx$

Example 2: Find the general (what does “general” mean here?) indefinite integrals:

(a) $\int (10x^4 - 2 \sec^2 x + \pi) dx$

(b) $\int (x+1)(1+2x^4) dx$

Example 3: Find the general indefinite integral:

(a) $\int \frac{\cos x}{1 - \cos^2 x} dx$

(b) $\int (x^2 + 2^x + 1) dx$

Example 4: Find the following indefinite integrals.

a) $\int \left(\frac{3-x}{x}\right)^2 dx$

b) $\int \left(\frac{x}{7} - \frac{7}{x}\right) dx$

Example 5: Evaluate the following integrals. Why is the $+C$ unnecessary here?

(a) $\int_0^9 \sqrt{2x} dx$

(b) $\int_0^1 x(\sqrt[4]{x^5} + \sqrt[5]{x}) dx$

Example 6: Evaluate the following integrals.

(a) $\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} dt$

(b) $\int_{\pi/4}^{\pi/2} \frac{1 + \sin^2 \theta}{\sin^2 \theta} d\theta$

The Net Change Theorem

The integral of a rate of change is the net change:

$$\int_a^b F'(x) dx = F(b) - F(a)$$

Examples of Physical Situations

- If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time t , what does $\int_{60}^{180} r(t) dt$ represent?
- A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t) dt$ represent?
- If $w'(t)$ is rate of growth of a child in pounds per year, what does $\int_0^5 w'(t) dt$ represent?
- If the units for x are feet and $a(x)$ are pounds per foot what are the units for the following?

(a) $\frac{da}{dx}$

(b) $\int_2^8 a(x) dx$

Example 7: The water flows from the bottom of a storage tank at a rate of $r(t) = 500 - 2t$ gallons per minute for $0 \leq t \leq 250$. Find the total amount of water that flows from the tank during the first hour.

Example 8: A particle moves along a line so that its velocity at time t is $v(t) = t^2 - 2t$ (measured in meters per second).

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.

(b) Find the distance traveled during this time period.